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Microeconomics (I)

Ch 5 More on Consumer Theory

Demand: price-quantity relationship from consumer's point of view

Price changes→ Quantity of demand changes

$P_X\uparrow \Rightarrow X^*\downarrow$?

Consumer uses her/his income to purchase X & Y

$P_X\uparrow \Rightarrow \frac{P_x}{P_y}\uparrow$ or X is relatively more expensive

$\rightarrow y^*$ (Y is relatively cheaper)

uses a cheaper good to substitute for X substitution effect

$P_X\uparrow \Rightarrow \frac{m^{\rightarrow \text{monetary income}}}{P_x}$ =>real income ↓

↑income in terms of good X(real income)

If X is normal=>X^* ↓

inferior=>X^* ↑ Income effect

P_X (P_y, m unchanged)
↗Substitution effect (SE) $\frac{P_X}{P_y}\uparrow \Rightarrow X^*\downarrow$
↘income effect $\frac{m}{P_X}\downarrow$ ↗$X^*\downarrow$ if X is normal; ↘$x^*\uparrow$ if X is inferior

$P_X\uparrow \Rightarrow X^*\downarrow$ if X is normal =>符合 law of demand

=>$X^*\downarrow$ if X is inferior
- $|SE| > |IE|$ => 符合 law of demand
- $|SE| < |IE|$ => *violates law of demand* =>Griffin good

SE+IE=Price effect(PE)

decomposition of price effect into substitution into substitution effect and income effect

Law of demand P_X changes ($P_X\uparrow$) =>Quantity demand of X (X^* at P)

($X^*\downarrow$) (PE)

↗SE$\frac{P_X}{P_y}$ ↑(given some fixed P_y) X is relatively more expensive

PE =>$x^*\downarrow$,$y^*\uparrow$

↘IE$\frac{m}{P_X}$ ↓(given some fixed m) real income ↓

=>$x^*\downarrow$(normal good)

$x^*\uparrow$(inferior good)

If good X is normal, P_X ↑=> $x^*\downarrow$

Inferior, P_X ↑ $\rightarrow x^*\downarrow$ if $|SE| > |IE|$
$\rightarrow x^*\uparrow$ if $|SE| < |IE|$

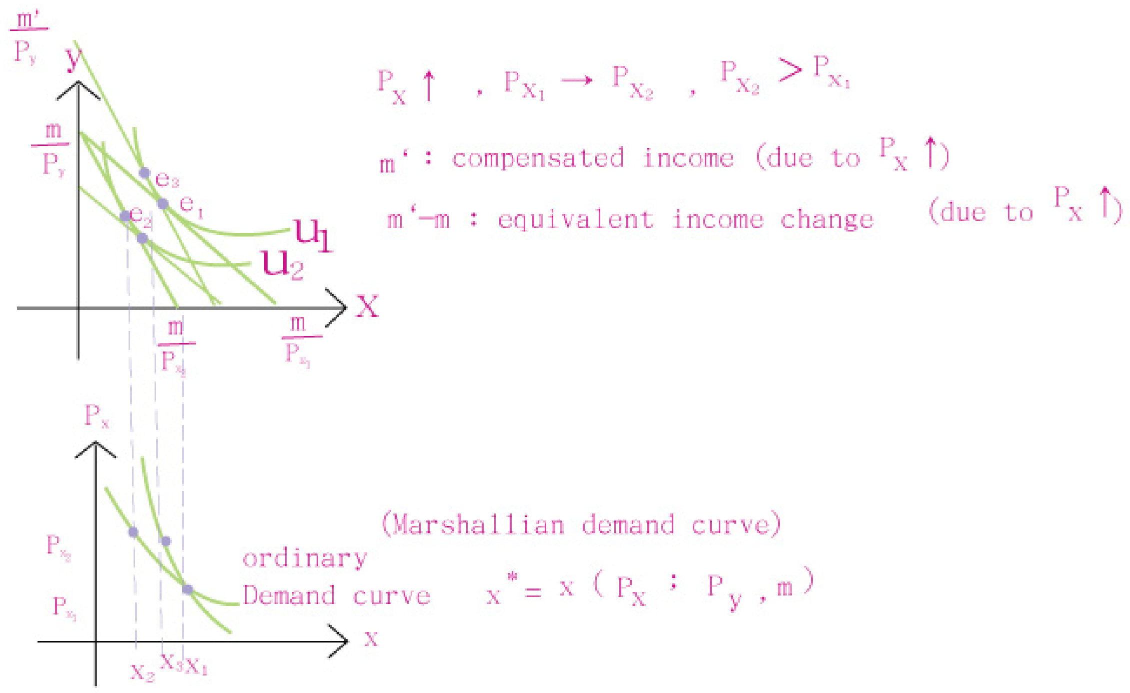
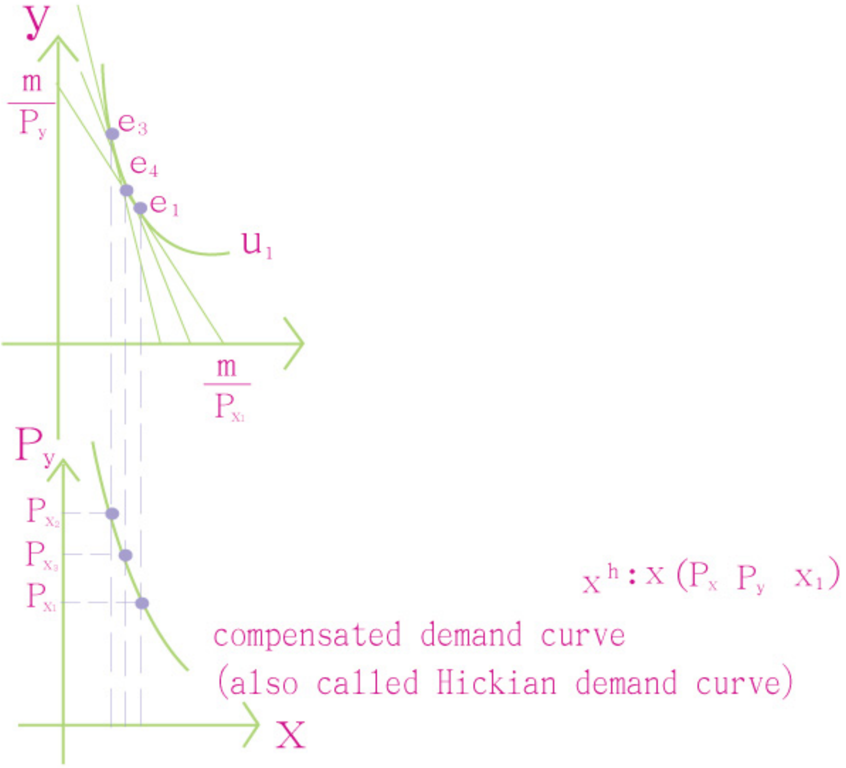
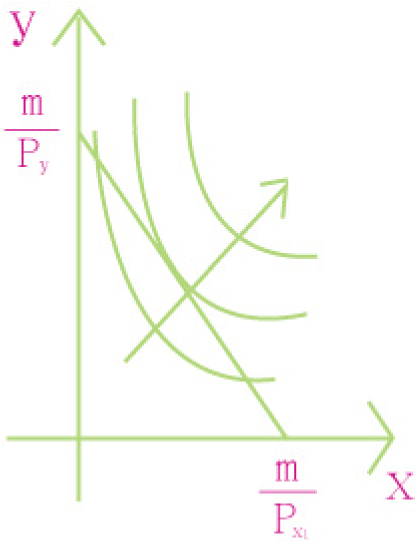


Figure57ordinary (Marshallian) Demand curve



Ｆigure58 :Compensated(Hickian) demand curve

$$\max_{x,y} \; u(x,y)$$



s.t. $P_X X + P_y Y = m$

Ｆigure59： Utility curve

FOC. $\begin{cases} MRS_{xy} = \frac{P_X}{P_y} \\ P_X X + P_y Y = m \end{cases}$

$\Rightarrow$ $x^* = (P_X, P_y, m)$

$y^* = (P_X, P_y, m)$ ordinary demand curve(Marshallian)

$\min_{x,y}$ $P_X X + P_y y$ expenditure or income needed

s.t. $u(x,y) = u$

FOC. $\begin{cases} MRS_{xy} = \frac{P_X}{P_y} \\ u(x,y) = u \end{cases}$

$\Rightarrow$ $X^h = (P_X, P_y, u)$

$y^h = (P_X, P_y, u)$

Lagrangian

$L(x,y,\mu) = (P_X X + P_y Y) + \mu(u - u(x,y))$

FOC $\frac{\partial L}{\partial x} = P_X - \mu\frac{\partial u(x,y)}{\partial x} = 0$---------------①

$\frac{\partial L}{\partial x} = P_y - \mu\frac{\partial u(x,y)}{\partial y} = 0$----------------②

① $P_X = \mu MUx$---------①'

② $P_y = \mu MUy$ ----------②'

$\frac{①'}{②'}$ $\frac{P_X}{P_y} = \frac{MUx}{MUy} MRS_{xy}$

} FOC

③ $u(x,y) = u$

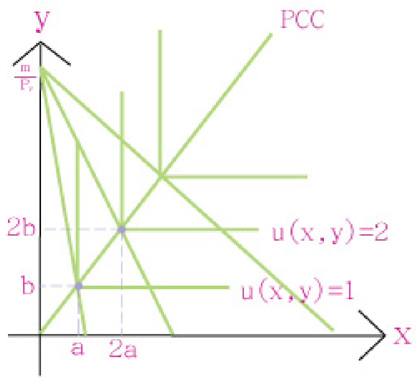


Figure60 :Utility curve that X&Y are perfect complements

(Example)

$X\&Y$ are perfect complements.

3 $u(x,y)=\min\{\frac{x}{a},\frac{y}{b}\}$

(x^*,y^*) must satisfy

$\frac{x}{a}=\frac{y}{b}$ at some kink point $\Rightarrow MRS_{xy}=\frac{P_X}{P_y}$ can't be used

$y=\frac{b}{a}x$ tangency condition

$P_X X+P_y Y=m \Rightarrow P_X X+P_y\frac{b}{a}X=m$

$aP_X X+P_y bx=am$

$x^*=\frac{am}{aPx+bPy}$ $x(P_X,P_y,m)$

Marshallian demand curve of X

$y^*=\frac{bm}{aPx+bPy}$ $y(P_X,P_y,m)$

Marshallian demand curve of Y

Compensated demand curve

$$x^h=X(P_X,P_y,u)=ua$$

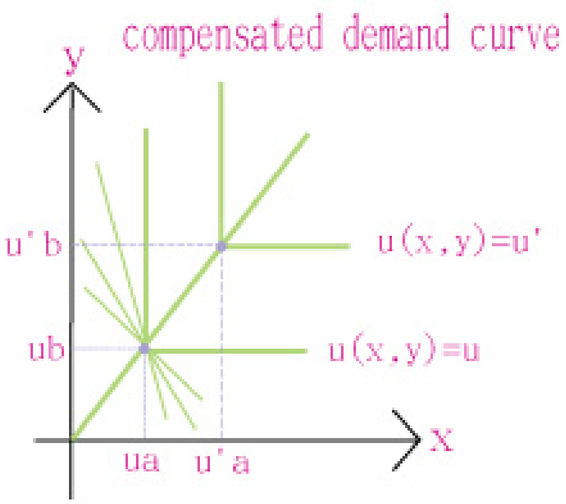


Figure61 :compensated demand curve

$$y^h = y\ (P_X, P_y, u) = ub$$

$$\frac{\partial X^*}{\partial P_X} = \frac{\partial \frac{am}{aPx + bP_y}}{\partial Px}$$

$$= -\frac{a^2 m}{(aP_x + bP_y)^2} < \quad 0$$

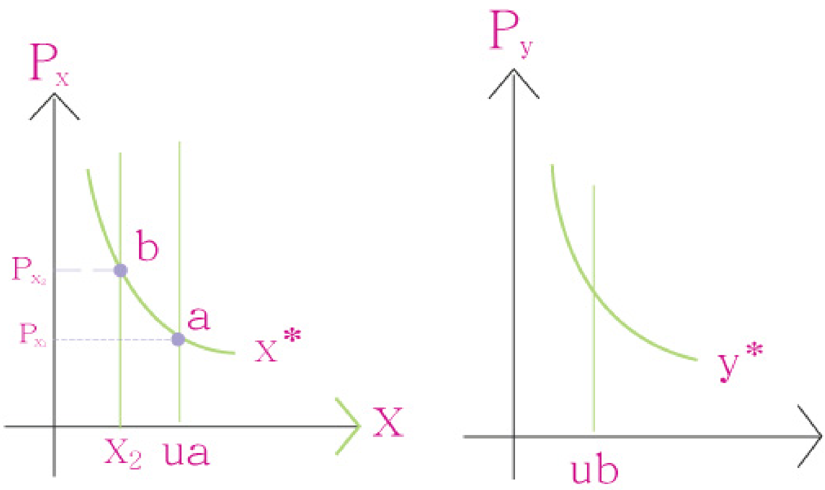


Figure62 :Demand curve

$$\frac{\partial^2 X^*}{\partial {P_x}^2} = \frac{a^3}{(aP_x + bP_y)^3} \quad > \quad 0$$

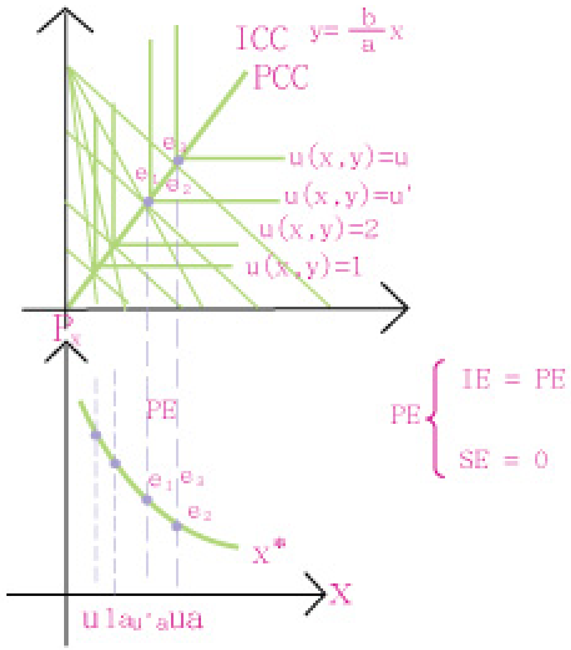


Figure63 :price effect (PE=IE)

(Example) $u(x,y)=\ln X+Y$

Ordinary demand curves

FOC $MRS_{xy} = \frac{P_X}{P_y}$--------------①

$P_X X+P_y=m$------------------②

① => $\frac{1}{x}= \frac{P_X}{P_y}$

=> $X^* = \frac{P_X}{P_y}$ independent of ② independent of m

② => $P_X \frac{P_y}{P_X} + P_y Y=m$

=> $y^* \frac{m}{P_y}$-1 independent of P_X

Compensated demand curve

FOC $MRS_{xy} = \frac{P_X}{P_y}$----------------③

$\ln x + y=u$ utility constraint

$MRS_{xy} \frac{MUx}{MUy} = \frac{1}{x}$

③ => $x^h = \frac{P_X}{P_y}$ independent of ④ independent of u

④ => $\ln \frac{P_X}{P_y} = u$

$y^h = u - \ln \frac{P_X}{P_y}$

$= u - \ln P_y + \ln P_x$

$P_X \downarrow$, $P_{X1} \rightarrow P_{X2}$, $P_{X2} < P_{X1}$

$e1 \rightarrow e2$

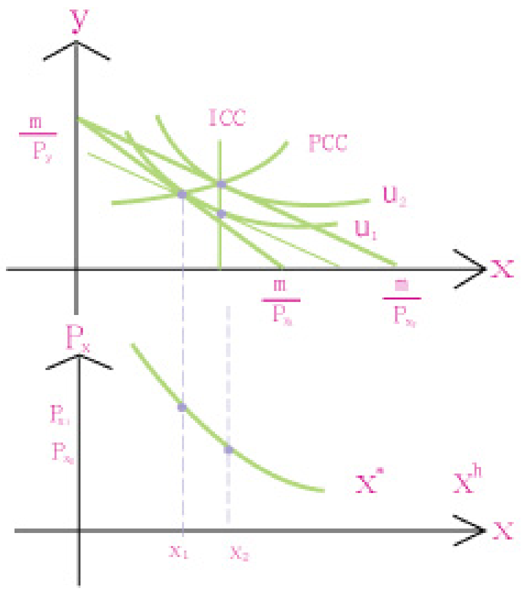


Figure64 :price effect(PE=SE)

$e1 \rightarrow \ e3$

$e3 \rightarrow \ e2$

PE= SE

only true for good X

(Example) Cobb-Douglas function

$u(x,y) \quad x^{\alpha} y^{\beta} \ \ , \quad \alpha, \beta > 0$

Ordinary demand curves

$$MRS_{xy} = \frac{P_X}{P_y} \text{-----------------①}$$

$$PxX+PyY=m\text{-----------------②}$$

$$MRS_{xy} = \frac{MUx}{MUy} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta}\frac{y}{x}$$

$$① \Rightarrow \frac{\alpha}{\beta}\frac{y}{x} = \frac{P_X}{P_y}$$

$$Y = \frac{\alpha}{\beta}\frac{y}{x}\ x \begin{pmatrix} \beta P_X X = \alpha P_y Y \\ P_y Y = \frac{\beta}{\alpha} P_X X \end{pmatrix}$$

$$② \Rightarrow P_X X + \frac{\beta}{\alpha} P_X X = m$$

$$\frac{\alpha+\beta}{\alpha} P_X X = m$$

$$\begin{cases} P_X X = \frac{\alpha}{\alpha+\beta}\, m & \text{expenditure on } x \\ P_y Y = \frac{\beta}{\alpha+\beta} m & \text{expenditure on } Y \end{cases}$$